Mathematics Methods 3 and 4

Test 2 Calculator Free

Name: ....

SHENTON COLLEGE

Teacher: Mrs Martin Dr Moore Mr Smith

Time Allowed : 30 minutes

Marks	/30

Materials allowed: Formulae Sheet provided.
Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

Question 1 **[4, 2, 2 = 8 marks]**

Determine the following:

(a) $\int \left(e^{2x} + \sqrt{x} + \pi\right) dx$

$= \frac{e^{2x}}{2} + \frac{2x^{\frac{3}{2}}}{3} + \pi x + c$ ✓ ✓ ✓ ✓

(b) $\frac{d}{dx}\int_x^3 \frac{t^2}{4}\,dt$

$-\frac{x^2}{4}$ ✓ ✓

(c) $\frac{d}{dx}\int_0^{x^2} \sqrt{1+t^2}\,dt$

$= \sqrt{1+x^4}\,(2x)$ ✓ ✓

Question 2 **[3, 4 = 7 marks]**

Evaluate

(a) $\int_0^2 \frac{3}{(2x+1)^4}\,dx = \int_0^2 3(2x+1)^{-4}\,dx$

$= \left[\frac{3(2x+1)^{-3}}{-6}\right]_0^2$ ✓ Antideriv Correct

$= \left[\frac{-1}{2(2x+1)^3}\right]_0^2$

$= \frac{-1}{250} - \frac{-1}{2}$ ✓ Sub boundaries

$= -\frac{1}{250} + \frac{125}{250}$

$= \frac{124}{250}$ ✓ $= \frac{62}{125}$

(b) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\sin 2x\,dx$

✓ $= \left[-\frac{\cancel{2}\cos 2x}{\cancel{2}}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

✓ $= -\cos 3\pi - (-\cos \pi)$

$= -(-1) + (-1)$ ✓ Correct exact values

$= 0$ ✓

$-1 \qquad -5$

$-3 \qquad 0 \qquad 2$

Question 3 **[1, 3 = 4 marks]**

Given $\int_0^{-3} f(x)dx = 1$ and $\int_0^{2} f(x)dx = -5$, find

(a) $\int_{-3}^{2} f(x)dx$

$= -6$ ✓

(b) $\int_0^{2} [3f(x) - 4]dx$

$= 3\int_0^2 f(x)dx - \int_0^2 4\,dx$ ✓

$= -15 - 8$ ✓

$= -23$ ✓

Question 4 **[5 marks]**

Given $\frac{dy}{dx} = ae^x + 1$ and when $x = 1$, $\frac{dy}{dx} = 3$ and $y = 2$

Find the value of y when $x = 0$.

$\frac{dy}{dx} = ae^x + 1$

$3 = ae + 1$

✓ $\frac{2}{e} = a$

$\frac{dy}{dx} = \frac{2}{e}e^x + 1$

$= 2e^{x-1} + 1$

$y = 2e^{x-1} + x + c$ ✓

$2 = 2e^0 + 1 + c$ ✓

$-1 = c$

$y = 2e^{x-1} + x - 1$ ✓

at $x = 0$ $\quad y = \frac{2}{e} - 1$ ✓

Question 5 **[1, 1, 2, 2 = 6 marks]**

Given $h(x) = \int_0^x \cos(2t)dt$, determine

a) $h'(x)$

$h'(x) = \cos 2x$ ✓

b) $h'\left(\frac{\pi}{2}\right)$

$h'\left(\frac{\pi}{2}\right) = \cos\left(2\frac{\pi}{2}\right) = -1$ ✓

c) $h\left(\frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \cos(2t)\,dt$

$= \left[\frac{\sin 2t}{2}\right]_0^{\frac{\pi}{2}}$ ✓

$\left(\frac{\pi}{2}, 0\right)$

$= \frac{\sin \pi}{2} - \frac{\sin 0}{2}$

$= 0 - 0 = 0$ ✓

d) the equation of the tangent to the curve $h(x)$ at $x = \frac{\pi}{2}$

$m = -1 \qquad \left(\frac{\pi}{2}, 0\right)$

$y = -x + c$ ✓

$0 = -\frac{\pi}{2} + c$

$\frac{\pi}{2} = c$

$y = -x + \frac{\pi}{2}$ ✓

Mathematics Methods 3 and 4

Test 2 Calculator Assumed

SHENTON COLLEGE

Name: ....

Teacher: Mrs Martin Dr Moore Mr Smith

Time Allowed: 20 minutes

Marks	/27

Materials allowed: Formulae Sheet provided. Classpad, calculators, 1 A4 page of notes, one side.
Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

Question 1 [2, 2, 2, 2 = 8 marks]

The acceleration (m/s^2) of a particle moving in a straight line is given by $a = 2t - 4$. The particle's initial velocity is 3 m/s. Its initial displacement from the origin is -15 m.

(a) Find the expression for the particle's velocity at any time.

$v(t) = t^2 - 4t + 3$ ✓ ✓

(b) Find the time(s), if any, when the particle comes to rest.

$t = 1$ ✓ and $t = 3$ s ✓

(c) Find its displacement when $t = 3$

$x(t) = \frac{t^3}{3} - 2t^2 + 3t - 15$ ✓

$x(3) = -15$ ✓ m

(d) Find the distance travelled in the first 3 seconds.

$\text{Dist} = \int_0^3 |t^2 - 4t + 3| \, dt$

$= \frac{8}{3}$ m ✓✓

Question 2 **[3, 6 = 9 marks]**

Consider the functions: $f(x) = x(5-x)$ and $g(x) = x(x-3)$

(a) Write down an integral which when evaluated will determine the area trapped between the two functions and calculate the area.

$$A = \int_0^4 x(5-x) - x(x-3)\,dx = 21\tfrac{1}{3} \text{ square units}$$

Limits

(b) Within the area trapped between the two functions a vertical line is drawn, intersecting $f(x)$ at Point P and intersecting $g(x)$ at Point Q.

(i) Show use of calculus to find the value of x for which the length of line segment PQ is a maximum.

Let L be the length

$$L = x(5-x) - x(x-3)$$

$$\frac{dL}{dx} = -4x + 8 = 0$$

$$x = 2$$

(ii) Use the second derivative test to show that this value of x does indeed produce a maximum value.

$$\frac{d^2L}{dx^2} = -4$$

For all values of x $\quad \frac{d^2L}{dx^2} < 0 \quad \therefore$ Concave down

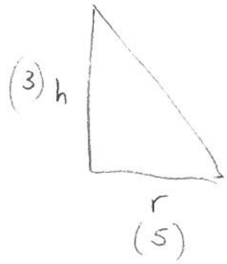
$\therefore$ Maximum

(iii) State the maximum length possible.

$L = 8$ units

Question 3 **[2, 3 = 5 marks]**

The ratio of the radius (r) to the height (h) is 5:3 for a specific cone.



(a) Show that the volume of the cone is given by $V = \dfrac{25\pi h^3}{27}$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{5h}{3}\right)^2 h$$

$$= \frac{25\pi h^3}{27}$$

$$r = \frac{5h}{3}$$

(b) Use the method of small change to find the approximate increase in the volume of the cone if the height changes from 5 cm to 5.02 cm.

$$\frac{dV}{dh} = \frac{25\pi h^2}{9} \qquad \delta h = 0.02$$

$$\delta V \approx \frac{dV}{dh} \times \delta h \approx \frac{25\pi(5)^2}{9} \times 0.02$$

$$\delta V \approx 4.4 \text{ cm}^3$$

Question 4 **[2, 3 = 5 marks]**

The cost, $C(x)$ (\$1000s) of manufacturing a product is given by $C(x) = 45 + 65x$. The revenue, $R(x)$, is given by the function $R(x) = 100x - 2.5x^2$. The manufacturer can only make between 2 and 10 products per week.

Find

(a) a simplified expression for the Profit if x units are made and sold.

$$P(x) = 100x - 2.5x^2 - (45 + 65x)$$

$$= 35x - 2.5x^2 - 45$$

(b) the minimum and maximum profit possible each week.

Min Profit \$15000

Max Profit \$77500

(in 1000s)